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## Statistical research and simulation of MEMS gyros measurements

*This article provides results of the statistical analysis and numerical evaluation of noise level components in the MEMS gyroscopes measurements. The Simulink-model of measurement errors for the ADIS16250 angular rate sensors was build, and test data and simulation results were analyzed.*

### Introduction

Strapdown inertial navigation is based on the use of inertial modules that produce the data used by the associated algorithms to get an idea about the current position of the object, its speed and orientation. The accuracy of above parameters directly depends on the accuracy of the sensors in use.

Traditional applications for inertial navigation in the Aerospace and Marine industry utilize the high precision sensors that are quite expensive to develop and produce. It is a trend of today to select the module components that include more energy efficient, more compact and inexpensive to produce sensors. These requirements correspond to the sensors built on micro-electro-mechanical systems (MEMS).

Design and production of sensors of this type have achieved great success, as evidenced by the mass production and variety of models. However, the miniaturization of MEMS sensors and production volumes do not allow them to achieve high precision in measurements. Their use often requires algorithmic additions to improve the accuracy of the data. This applies particularly to MEMS-gyros. In this paper, we study the statistical properties of the measurements of the Analog Devices' MEMS gyroscopes ADIS16250. Technical documentation, which is available on the website of the Analog Devices, does not provide complete information about individual characteristics of a single sensor. This creates the need for additional research.

### Objective.

The aim of this work is to identify the noise from each MEMS-gyro included into assembly. This task includes processing of the measurements, getting coefficients for Allan variance and computer simulation. Hereafter, the information about the noise level makes it possible to adjust the obtained measurements.

Mathematical error model including both systematic and random components is needed for algorithmic data correction. At all, components of the error model can be divided into additive (zero offset, drift, casual care zero) and multiplicative (the error of scale factor linearity, asymmetry). The additive error of the gyro zero signal  $\omega_0$  makes the greatest contribution into the final error of gyro; it is represented in the form of several components

$$\omega_0 = \omega_{\text{сучм}} + \omega_{\text{заг}} + \omega_{\text{случ}} \quad (1)$$

where  $\omega_{cucm}$  is the systematic component that does not depend of external factors and keeps its value within the service life;  $\omega_{зав}$  – gyro drift component depending on the temperature, storage time, overload, current time in operation and other factors;  $\omega_{случ}$  – random noise component of the zero signal.

Working model of the system is an inertial measurement unit on a fixed base with a control and computer interface PCB (Fig. 1). It includes three integrated MEMS-gyro ADIS 16250.

During of the experiment, records were made under static conditions and constant temperature. Data recording occurred with a frequency of 50 Hz, the duration of the session – 25 minutes. The signal of the gyroscope, taken at a static position shown in Fig. 2.

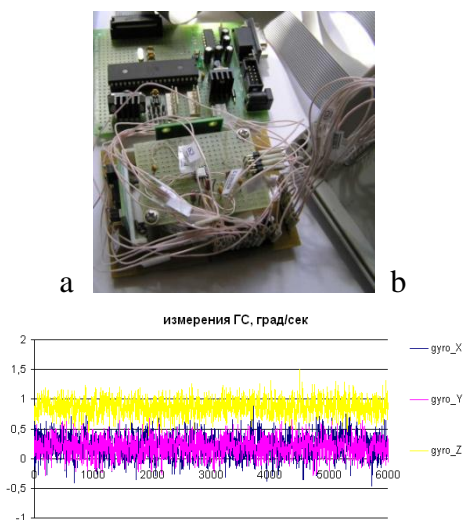


Fig. 1. a – Inertial measurement unit built on MEMS-sensors; b – measurements from 3 MEMS-gyros.

The most of theoretical conclusions and practical results is carried out in the literature for stationary series; so of course, the first task is to study the stationarity of the available data.

First, the conclusion about the stationarity of the measurements can be based on the variation of the average value and dispersion value within a time period. The moving average method is selected for averaging. See Fig. 2 that shows plots of the velocity along the X-axis at various parameters of the averaging. The relationship between the average value and time indicates the presence of zero drift and trend. Stationary time series were obtained with the help of the linear regression and

subsequent data filtering. The mean value and variance value of the time series do not change over time and therefore, the process can be considered stationary.

Another approach to the stationarity study is based on the comparison between the values of distribution density function for samples obtained by splitting a primary time series into equal sequential intervals. Analysis of the density distribution functions obtained for samples from one implementation, and different implementations, shows comparable values (Fig. 2 b). Thus, stationarity of the time series registered in the experiment is determined.

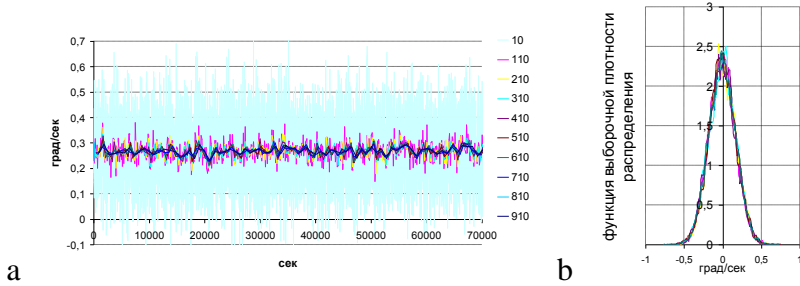


Fig. 2 – Stationarity of the samples: a - measurement of average value after averaging, b) selective distribution.

Thus, the values of the zero offsets of the sensors are (grad./sec.): 0,0104009 for the X axis - 0,000995936 the Y-axis 0,00746764 axis Z. These values define the first component in (1).

The second component  $\omega_{\text{заг}}$  can be defined as formula connection of the gyro error from design features and external factors obtained from the physical principles of device operation.

Regarding the third component in (1), traditional methods of spectral analysis did not answer the question about the structure of the noise available in the measurements and their intensity. Currently, Allan variation [1] starts to find a wide use as a method for estimating the noise component.

$$\sigma_A^2(\tau) \approx R^2 \frac{\tau^2}{2} + K^2 \frac{\tau}{3} + B^2 \frac{2}{\pi} \ln 2 + N^2 \frac{1}{\tau} + Q^2 \frac{3}{\tau^2}, \quad (2)$$

where R, K, B, N, Q – typical noise components coefficients [2]. Namely, the R – linear change of the angular velocity, K is a random deviation of the angular velocity, B is the coefficient of instability of the zero signal, N is the coefficient of random angle deviation, Q – quantization noise factor.

Fig. 3 a, b, C shows the curves of Allan variation, based on the measurements for the three MEMS gyroscopes.

Comparing the figures, we note the following types of noise in the measurements: angle increment noise (N), instability of the zero offset (B) and angular velocity deviation noise (K). See Table 1 for the calculated values of the coefficients (2) for each sensitivity axis.

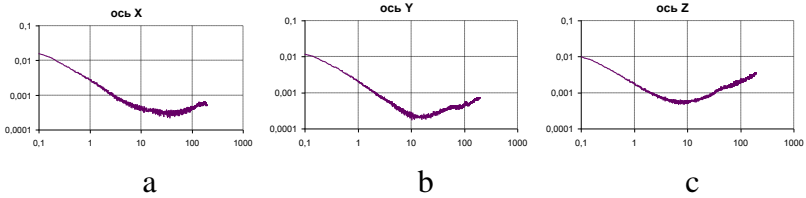


Fig. 3 –Allan variance graph: a) for the sensitivity axis OX, b) for axis OY, c) for axis OZ.

*Table 1*

Coefficients of Allan variation curves

The direction of sensitivity axis in the assembly	Coefficients of polynomial (2)				
	$ R^2 , \times 10^{-8}$ M/c <sup>3</sup>	$ K^2 , \times 10^{-5}$ M/c <sup>2</sup> /√c	$ B^2 , \times 10^{-4}$ M/c <sup>2</sup>	$ N^2 , \times 10^{-3}$ M/c/√c	$ Q^2 , \times 10^{-5}$ M/c
X	1,336	1,176	2,074	2,662	3,65
Y	2,352	1,754	0,0106	2,115	3,116
Z	5,839	6,607	5,135	1,59	1,962

Based on this analysis, errors model for ADIS 16250 MEMS gyro is as follows:

$$\omega_g = \omega + b + n_a, \dot{b} = n_b \quad (3)$$

where  $\omega_g$  - simulated output signal of the sensor;  $\omega$  - true angular velocity;  $b$  - zero drift of the gyro (rad/s) imposed by the random drift of the angular velocity  $n_b$ ;  $n_a$  - white noise, distorting the signal of the angular velocity measured by the gyroscope, which determines the random drift when measuring angle as per the gyroscope data.

Simulink model (Fig. 4) was compiled with the help of the error model specified in (3) and simulation exercise was conducted with the results provided in Fig. 5. Comparison of the real signal graph (Fig. 5a) and simulated signal graph (Fig. 5 b) confirms that the approach specified in this article can be used to identify noise of MEMS sensors.

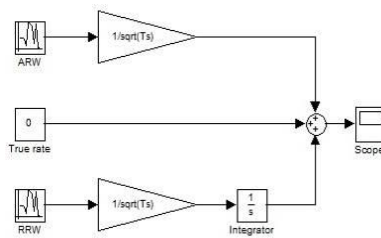


Fig. 4. Simulink model of ADIS16250 angular rate sensor measurement errors

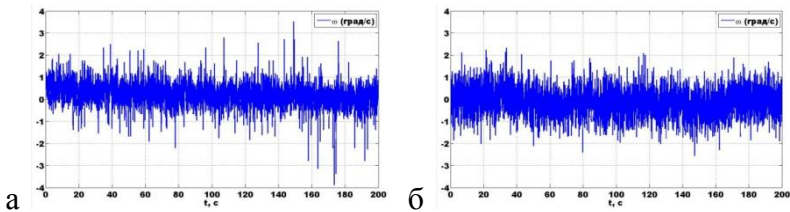


Fig. 5. Gyro signal: (a) experimental; b) simulated

### Conclusions.

This paper presents the results of the statistical analysis for MEMS gyroscopes measurements. Evidences for the stationarity of the time series realizations and numerical estimates of the level of the inertial measurements noise components were obtained. The obtained noise component estimates were compared to the claimed characteristics from the manufacturer. The Simulink model of angular rate measurement errors, which allows to recover the real signal, was built for the ADIS16250 sensors that are included into the inertial measurement unit.

### References

1. Allan D.W. Statistics of atomic frequency standards // Proc. IEEE. – Vol. 54, N2
2. IEEE Std 1554-2005 IEEE Recommended Practice for Inertial Sensor Test Equipment, Instrumentation, Data Acquisition, and Analysis.